

Abstract

Piloting difficulties associated with conducting maneuvers in hypersonic flight are caused in part by the nonintuitive nature of the aircraft response and the stringent constraints anticipated on allowable angle-of-attack and dynamic pressure variations. This report documents an approach that provides precise, coordinated maneuver control during excursions from a hypersonic cruise flight path and observes the necessary flight condition constraints. The approach is to achieve specified guidance commands by resolving altitude and cross-range errors into a load factor and bank angle command by using a coordinate transformation that acts as an interface between outer- and inner-loop flight controls. This interface, referred to as a “resolver,” applies constraints on angle-of-attack and dynamic pressure perturbations while prioritizing altitude regulation over cross range. An unpiloted test simulation, in which the resolver was used to drive inner-loop flight controls, produced time histories of responses to guidance commands and atmospheric disturbances at Mach numbers of 6, 10, 15, and 20. These time histories are used to illustrate the manner in which the overall control system incorporates flight condition constraints and accounts for high-speed flight effects. Angle-of-attack and throttle perturbation constraints, combined with high-speed flight effects and the desire to maintain constant dynamic pressure, significantly impact the maneuver envelope for a hypersonic vehicle. Turn-rate, climb-rate, and descent-rate limits can be determined from these constraints. Density variation with altitude strongly influences climb- and descent-rate limits and throttle modulation if dynamic pressure is to be maintained during vertical transitions between cruise flight conditions.

Introduction

Many recently proposed hypersonic vehicle concepts present significant problems in guidance, flight control, and flying qualities that must be addressed if such designs are to be made viable. These air-breathing single-stage-to-orbit (SSTO) vehicle designs traverse a broader range of flight regimes than aircraft that have flown in the past, and must emphasize performance during all phases of flight from takeoff to orbit to achieve their mission. Stringent constraints on angle-of-attack, sideslip, and dynamic pressure variations must be observed, since hypersonic propulsion system performance is strongly dependent on flight condition and may suffer dramatically if excessive variation is experienced. Also, an unprecedented level of coupling exists between flight dynamic, propulsion, and structural modes, and the

degree of this coupling may vary significantly over such a large design envelope. The anticipated sensitivity of the propulsion system to variations in flight conditions makes precise regulation of angle of attack, angle of sideslip, dynamic pressure, and flight path imperative, while the high degree of coupling makes such tight control difficult to achieve. The area of maneuver control for hypersonic vehicles requires further investigation to reveal the various piloting issues and to determine the level of automation necessary to achieve the desired performance.

Previous work by Berry (ref. 1) involved a piloted simulation of a hypersonic configuration in which numerous issues in handling qualities relevant to maneuvering in hypersonic flight were examined. Recent research by Lallman and Raney (refs. 2 to 6) has been focused on regulation of trajectory parameters during hypersonic maneuvers at cruise flight conditions, but maneuvers were restricted to the vertical plane. This report is focused on the problem of coordinated control of maneuvers in both the vertical and lateral planes at hypersonic cruise flight conditions. An approach is presented that provides maneuver coordination through the resolution of altitude and cross-range errors into a combination of normal load factor and bank angle command. The goal of this research is to define an automatic control design option for executing coordinated maneuvers in hypersonic flight while regulating key parameters, such as angle of attack, angle of sideslip, and dynamic pressure. Additionally, this work is intended to be an investigation of the manner in which propulsion and flight condition constraints impact the maneuvering capabilities of hypersonic aerospacecraft.

A control integration concept is described in this report for powered hypersonic cruise-turn maneuvers. First, several introductory remarks on critical issues relevant to the control of air-breathing vehicles in hypersonic flight are made. An overview of the basic components of a conceptual control system for an air-breathing hypersonic vehicle is then presented. The overall control system is divided into three main subsections consisting of (1) inner-loop controls to provide stability augmentation, (2) outer-loop controls to track guidance commands and reject disturbances, and (3) an interface between the inner and outer loops that resolves vertical and lateral acceleration commands into a lift vector command that is specified by a normal load factor and bank angle combination. Each of these subsections is described in order following the control system overview.

The interface between inner and outer loops, referred to throughout this report as a “resolver,” is the main subject of this research. The resolver concept

is introduced following the discussion of the outer-loop controls. Logic associated with the resolver to prioritize regulated trajectory parameters and incorporate flight condition constraints is then developed. Following this description of the resolver, a throttle control law, which regulates dynamic pressure during hypersonic maneuvers, is presented. The manner in which constraints on angle-of-attack and thrust perturbations impact the maneuver envelope of a hypersonic vehicle is also discussed. A control design case is presented for an example vehicle, and a test simulation is described which was used to produce time histories of responses to guidance commands and atmospheric disturbances. These responses illustrate the manner in which the combination of inner-loop, resolver, and outer-loop control systems operate. The simulated responses are discussed, and several conclusions are presented that have significant implications for maneuver control during hypersonic flight.

Symbols

A_Y	lateral acceleration in body-axis coordinate system, ft/sec ²	$K a_{\dot{\phi}}$	roll-rate feedback gain to aileron
b	wing span, ft	$K r_{\beta}$	sideslip feedback gain to rudder
C_D	drag coefficient	$K r_r$	yaw-rate feedback gain to rudder
$C_{D,\alpha} = \frac{\partial C_D}{\partial \alpha}$		K_h	feedback gain for proportional altitude error
$C_{D,\alpha}^2 = \frac{\partial^2 C_D}{\partial \alpha^2}$		K_{hd}	feedback gain for derivative altitude error
C_L	lift coefficient	K_{hI}	feedback gain for integral altitude error
$C_{L,\alpha} = \frac{\partial C_L}{\partial \alpha}$		$K_{\bar{q}}$	feedback gain for dynamic pressure error
C_{ρ}	rate of density variation with altitude, slugs/ft ³ /ft	K_y	feedback gain for proportional cross-range error
D	drag force, lb	K_{yd}	feedback gain for derivative cross-range error
g	acceleration due to gravity, ft/sec ²	K_{yI}	feedback gain for integral cross-range error
h	altitude, ft	L, M, N	rolling, pitching, and yawing moments, ft-lb
I_x	rolling moment of inertia, slugs-ft ²	Mach	ratio of vehicle velocity (relative to air mass) to speed of sound
I_{xz}	product of inertia, slugs-ft ²	m	mass, slugs
I_z	yawing moment of inertia, slugs-ft ²	n	normal load factor with respect to vehicle flight path, g units ($1g = 32.17$ ft/sec ²)
$K a_{\beta}$	sideslip feedback gain to aileron	n_{ℓ}	lateral component of normal load factor with respect to an Earth-fixed axis system, g units
$K a_{\delta_r}$	rudder deflection cross-feed gain to aileron	n_v	vertical component of normal load factor with respect to an Earth-fixed axis system, g units
$K a_{\phi}$	bank angle feedback gain to aileron	p, q, r	roll, pitch, and yaw rates in body-axis coordinate system, deg/sec
		\bar{q}	dynamic pressure, lb/ft ²
		R	altitude measured to center of Earth, ft
		S	wing area, ft ²
		s	Laplace transform parameter
		T	thrust force, lb
		u, v, w	body-axis velocities, ft/sec
		V	inertial velocity, ft/sec

W	weight, lb
X, Y, Z	force components of body axes, lb
x_b, y_b, z_b	vehicle body axes
y	cross range, ft
α	angle of attack, deg
β	sideslip angle, deg
Δ	perturbation value or deviation from nominal
δ_a	aileron deflection, positive right trailing edge down, deg
δ_r	rudder deflection, deg
ζ	damping ratio
θ	pitch angle, deg
λ	distance from center of gravity to lateral accelerometer, positive forward, ft
ρ	air density, slugs/ft ³
σ	real root in complex plane
τ	complementary filter time constant
ϕ	roll angle, deg
ψ	heading angle, deg
ω	natural frequency, rad/sec

Abbreviations:

APAS	aerodynamic preliminary analysis system
B-T-D	bank-to-dive
p-i-d	proportional-integral-derivative
POST	program to optimize simulated trajectories

Superscripts:

\cdot	first derivative with respect to time
$\ddot{}$	second derivative with respect to time
$\hat{}$	estimated value
\sim	complementary filtered value

Subscripts:

a	parameter used in aileron control law
acc	accelerometer signal
cmd	commanded value
err	error signal
h	parameter used in altitude regulator
max	maximum allowable value
min	minimum allowable value
prev	value from previous time step
r	parameter used in rudder control law
t	parameter used in throttle control law
y	parameter used in cross-range regulator
0	nominal or trim value

Dimensional stability and control derivatives:

$$\begin{aligned}
 L_\beta &= \frac{\bar{q}Sb}{I_x}C_{l_\beta} & N_\beta &= \frac{\bar{q}Sb}{I_z}C_{n_\beta} & Y_\beta &= \frac{\bar{q}S}{mV}C_{y_\beta} \\
 L_p &= \frac{\bar{q}Sb^2}{2VI_x}C_{l_p} & N_p &= \frac{\bar{q}Sb^2}{2VI_z}C_{n_p} & Y_p &= \frac{\bar{q}Sb}{2mV^2}C_{y_p} \\
 L_r &= \frac{\bar{q}Sb^2}{2VI_x}C_{l_r} & N_r &= \frac{\bar{q}Sb^2}{2VI_z}C_{n_r} & Y_r &= \frac{\bar{q}Sb}{2mV^2}C_{y_r} \\
 L_{\delta_a} &= \frac{\bar{q}Sb}{I_x}C_{l_{\delta_a}} & N_{\delta_a} &= \frac{\bar{q}Sb}{I_z}C_{n_{\delta_a}} & Y_{\delta_a} &= \frac{\bar{q}S}{mV}C_{y_{\delta_a}} \\
 L_{\delta_r} &= \frac{\bar{q}Sb}{I_x}C_{l_{\delta_r}} & N_{\delta_r} &= \frac{\bar{q}Sb}{I_z}C_{n_{\delta_r}} & Y_{\delta_r} &= \frac{\bar{q}S}{mV}C_{y_{\delta_r}}
 \end{aligned}$$

where $C_{\varepsilon_\eta} = \frac{\partial C_\varepsilon}{\partial \eta}$ for $\varepsilon = l, n$, or y and $\eta = \beta, \delta_a$, or δ_r and where $C_{\varepsilon_\eta} = \frac{\partial C_\varepsilon}{\partial \eta \frac{b}{2V}}$ for $\eta = p$ or r . The coordinate-system notation and reference frames used in this report are shown in figure 1.

Specialized terms:

- altitude priority: the practice of assigning priority to the elimination of altitude errors over the elimination of cross-range errors
- ballistic origin: origin of resolver coordinate system that designates the condition at which normal load factor is zero
- bank-to-dive: the practice of rolling the lift vector off of vertical to achieve the desired descent rate while remaining above some minimum angle of attack

centrifugal relief: reduction in normal load factor required to maintain constant altitude at hypersonic speeds that results from the centripetal acceleration of the vehicle as it circles the Earth

complementary filter: a low-pass filter that is augmented with derivative feedback in such a way as to eliminate its effect on closed-loop stability while attenuating responses to atmospheric disturbances

resolver: a rectangular-to-polar coordinate transformation that converts vertical and lateral acceleration commands into a load factor and bank angle command vector while applying limits and logic to incorporate features that limit altitude priority, “bank-to-dive,” and load factor

Issues Relevant to Hypersonic Flight Control

Several factors complicate maneuver control for air-breathing vehicles at hypersonic speeds. For example, strict regulation of dynamic pressure may be required to meet the constraints of a given ascent trajectory. This practice is particularly critical in the case of air-breathing SSTO vehicles, which may follow a trajectory profile determined by a combination of engine performance and heating criteria. At hypersonic speeds, dynamic pressure is strongly influenced by the variation of air density with altitude. Compensation for this effect during vertical maneuvers will almost certainly require some form of automatic feedback regulator, since it would otherwise represent a formidable increase in manual work load.

Also, lateral maneuvering in hypersonic flight is complicated by a high-speed flight effect that manifests itself as a reduction in load factor required to maintain constant altitude. A vehicle in hypersonic flight may approach orbital velocities while still in the atmosphere. The centripetal acceleration of the vehicle as it circles the Earth then becomes significant enough to cause a noticeable reduction in the load factor required to maintain level flight. This effect, referred to as centrifugal relief, can have an appreciable impact on the bank angle required to complete a given lateral maneuver. For instance, a conventional aircraft in level subsonic flight that is executing a coordinated turn with a bank angle of 60° experiences a normal acceleration of $2g$ units. However, a vehicle in hypersonic flight with a velocity of 18 500 ft/sec (about Mach 18 at 95 000 ft) is required to bank approximately 75° to produce a $2g$ turn (ref. 7). In straight and level flight about a spherical Earth, the normal load factor n required to maintain constant

altitude is

$$n = 1 - \frac{V^2}{Rg} \quad (1)$$

where V is the inertial velocity of the vehicle, R is its distance from the center of the Earth, and g is acceleration due to gravity. In subsonic flight, the term V^2/Rg is negligible, but at hypersonic speeds the term becomes significant. The influence of this term becomes more pronounced as the velocity of the vehicle approaches orbital speeds.

Another factor in hypersonic flight is the extremely slow response of trajectory variables such as heading or flight-path angle. Relatively long time periods are required to redirect the velocity vector, since the momentum of the vehicle is so great. Some form of predictive flight director may be necessary to guide the pilot through sustained and somewhat nonintuitive maneuvers. In addition to the complications that are inherent in maneuvering in the hypersonic regime, the current military specification manuals (Mil Specs) cannot provide adequate design criteria for the synthesis of a fly-by-wire system for hypersonic air-breathing vehicles. Shortcomings have been identified in the areas of roll control for hypersonic turns, response to atmospheric perturbations, cockpit displays for hypersonic maneuvering, and many other critical flight-control criteria (refs. 7 to 9). Also, the exact role of the pilot during the ascent portion of a transatmospheric flight is uncertain. Hypersonic propulsion modules are likely to be sensitive to angle-of-attack and angle-of-sideslip perturbations, which may disturb inlet conditions and result in poor engine performance (ref. 10). Such transients in inlet conditions could also result from responses to abrupt guidance commands. Extremely low tolerance to perturbations from the nominal flight condition may require that some tasks be automated if the desired mission is to be achieved.

The control design presented in this paper addresses several issues that are of particular significance to maneuvering in hypersonic flight. The design concept presented could be used to control the vehicle directly or to drive a cockpit-display flight director based on inputs from an onboard guidance algorithm. The concept acts as an interface between the onboard guidance system and the inner-loop flight controls.

An overview of the basic components of a conceptual control system for an air-breathing hypersonic vehicle follows, after which each subsection of the control system is described. Following the control system description, a design case is demonstrated for an example vehicle, and a test simulation is presented. Simulated responses to guidance commands

and atmospheric disturbances are discussed and are followed by several concluding remarks relevant to the control of air-breathing vehicles in hypersonic flight.

Control System Overview

The basic elements of the hypersonic control system concept are presented in figure 2. The overall control system is divided into three main subsections: (1) inner-loop controls to provide stability augmentation, (2) outer-loop controls to track guidance commands and reject disturbances, and (3) an interface between the inner and outer loops, which resolves vertical and lateral acceleration commands into a normal load factor and bank angle command vector. A summary of the overall system is presented in this section, and each element is described in detail in subsequent sections.

An onboard guidance system is envisioned that will issue commands in terms of altitude and cross range to achieve a given task, such as executing a heading change or eliminating vertical and lateral offsets from a desired trajectory profile. These commands are differenced from actual feedback values, as shown in figure 2, to produce error signals. Altitude and cross-range control laws that translate errors from the desired trajectory parameters into vertical and lateral acceleration commands are then applied. This feedback scheme makes up the outer-loop control system, which consists of two separate regulator loops to track altitude and cross-range guidance commands. The specific architectures of the outer-loop altitude and cross-range control laws are described subsequently in this report.

Vertical and lateral acceleration commands from the outer loops are converted into load factor components that take into account the reduction in vertical load factor that is caused by centrifugal relief. These load factor commands are then fed into a rectangular-to-polar coordinate transformation, which converts the vertical and lateral load factor components into a normal load factor and bank angle command vector (fig. 2). Logic and limiting strategies are associated with the coordinate transformation to incorporate various flight condition constraints and to prioritize the regulation of trajectory parameters. The coordinate transformation and its associated limits and logic are referred to as a resolver throughout this report. This resolver acts as an interface between the outer loops, which track guidance commands, and inner loops, which provide stability augmentation.

In this study, the load factor and bank angle command vectors produced by the resolver were fed

directly into an inner-loop flight controller. (In a real-time investigation, the outputs from the resolver might have been used to drive a cockpit display to aid the pilot in conducting the desired maneuver.) A simple inner-loop controller is presented in this report to allow simulation of the completed system, although more sophisticated inner-loop designs can be used with the resolver strategy. The inner-loop controller commands rudder deflections, elevon deflections, and thrust variations to track the load factor and bank angle command vectors from the resolver while maintaining constant dynamic pressure during a maneuver. The inner-loop control design is parameterized in terms of specified frequency and damping for the desired response characteristic and requires values for the aerodynamic stability and control coefficients of the vehicle.

Inner-Loop Flight Controls

A simple inner-loop controller was developed to allow simulation of the overall control concept. Only lateral flight-control laws were designed, and perfect pitch control was assumed for the test simulation. Short-period dynamics and propulsive thrust and moment variations with angle of attack were not considered in this inner-loop design. Inclusion of these effects would not invalidate the resolver design concept presented in this report, but it would necessitate a more elaborate inner-loop controls design. The inner-loop controller acts to stabilize the lateral flight dynamic modes, which provides acceptable frequency and damping characteristics while following commands issued by the resolver interface with the outer loop. It is crucial that the angle of sideslip be kept to nearly zero during any maneuvers, so that engine inlet tolerances are not violated and so that excessive drag is not produced (i.e., the maneuvers should be coordinated).

The approach used in this study has been to develop two classical feedback loops to drive the ailerons and rudder. The aileron control loop tracks bank angle commands that are issued by the resolver, while the rudder control loop maintains zero sideslip throughout the maneuver. The effect of rudder deflection on rolling motion is compensated by a cross-feed to the aileron control law. A simple pole-placement technique is used in each of the two control loops to determine the required feedback gains. Desired inner-loop dynamics are specified in terms of frequency and damping, which allows the designer to place the closed-loop poles at a location that might be desirable from a handling-qualities perspective. Derivations of the rudder and aileron control laws are presented in appendix A.

The feedbacks used in the rudder control law are yaw rate and sideslip angle, while those used in the aileron control law are roll rate, bank angle, and sideslip angle. Angle of sideslip is an important measurement that appears in both these control laws. However, the reliability of air-data measurements during some phases of hypersonic flight is uncertain. In some cases, it may be necessary to augment traditional air-data sensors with an estimate of air-data measurements based on inertial instruments. For demonstration purposes, the inner-loop control system designed in this report utilizes an angle-of-sideslip feedback that is constructed from a lateral-accelerometer reading rather than an air-data sensor array. The lateral accelerometer is placed at the instantaneous center of rotation in response to a rudder deflection; this placement eliminates the need to correct the accelerometer reading for rudder effects. A detailed description of the angle-of-sideslip estimator is included with the formulation of inner-loop controls in appendix A. The use of air-data estimates based on the designer's knowledge of the aerodynamic model makes the control system sensitive to the fidelity of the aerodynamic predictions. Such air-data estimates may be included in a weighting scheme in which measurements from various sensors are combined to derive a best estimate of the quantity in question.

Additionally, care must be taken to avoid feeding crosswind gusts directly into the lateral control laws via sideslip feedback. High-speed flight through atmospheric disturbances could otherwise cause sharp peaks in commanded surface deflections. Since extremely sharp peaks in the commanded position of control surfaces are generally undesirable and tend to cause actuator-rate saturation, a low-pass filter is added to the sideslip-angle feedback loop. The filter is complemented with derivative feedback to eliminate its effect on the stability of the closed loop and to prevent it from interfering with the response to pilot or guidance system commands. Because of the derivative feedback through the filter, this network is referred to as a "complementary filter." The complementary filter has a unity transfer function and does not affect the stability of the closed-loop system. The filter acts only to reduce high-frequency control action in response to atmospheric disturbance inputs, and not to commanded control inputs. It would also be necessary to filter the signal appropriately to eliminate corruption of the measurement as a result of structural vibrations if a flexible vehicle model were used. A formulation of the complementary filter is included in appendix A. Several factors are also included in this appendix that reflect important consid-

erations in inner-loop controls design for hypersonic vehicles. Although a relatively simple inner-loop design technique has been used for this investigation, more sophisticated designs can also be used in conjunction with the resolver concept presented in this report.

Outer-Loop Flight Controls

The outer loops of the control system shown in figure 2 track guidance commands in altitude and cross range. They convert vertical and lateral position errors into vertical and lateral acceleration commands. For the outer loops, a proportional-integral structure with derivative feedback is used to provide damping (p-i-d control). A dual-loop architecture that regulates altitude and cross range is used to provide coordinated tracking of maneuver commands from the guidance system. The outer loops are synthesized separately, but their command signals to the inner-loop controls are coupled through the action of the resolver presented subsequently in this report. Inner-loop dynamics are assumed to be sufficiently fast so as not to invalidate the integrated system design. This is not an unreasonable assumption, since trajectory parameters are slow to respond in hypersonic flight. Figure 3 is a block diagram of the basic altitude loop architecture. The feedback gains in the p-i-d control loop may be specified in terms of frequency and damping as described below.

The inner loop for vertical acceleration response is assumed to operate with sufficient speed to be transparent to the outer-loop dynamics, so its transfer function is represented as a gain of 1. The closed-loop transfer function for the altitude loop shown in figure 3 may then be expressed as

$$\frac{h}{h_{cmd}} = \frac{K_{hd}K_h(s + K_{hI})}{s^3 + K_{hd}s^2 + K_{hd}K_hs + K_{hd}K_hK_{hI}} \quad (2)$$

The third-order characteristic equation of this transfer function can be represented in terms of a product of first- and second-order expressions as

$$(s^2 + 2\zeta_h\omega_hs + \omega_h^2)(s + \sigma_h) = 0 \quad (3)$$

or

$$s^3 + (\sigma_h + 2\zeta_h\omega_h)s^2 + (2\zeta_h\omega_h\sigma_h + \omega_h^2)s + \omega_h^2\sigma_h = 0 \quad (4)$$

where ω_h and ζ_h represent frequency and damping of the harmonic response, and σ_h is the real root associated with the exponential portion of the response.

Feedback gains K_{hd} , K_h , and K_{hI} for the p-i-d controller depicted in figure 3 are now solved in terms of frequency and damping of the second-order characteristic, and the real first-order root, by equating coefficients in equation (4) with coefficients in the denominator of equation (2) as follows:

$$K_{hd} = \sigma_h + 2\zeta_h\omega_h \quad (5a)$$

$$K_h = \frac{\omega_h^2 + 2\zeta_h\omega_h\sigma_h}{\sigma_h + 2\zeta_h\omega_h} \quad (5b)$$

$$K_{hI} = \frac{\omega_h\sigma_h}{\omega_h + 2\zeta_h\sigma_h} \quad (5c)$$

The lateral loop has an identical p-i-d architecture, and the feedback gains are determined by analogous equations; the subscript h is replaced by the subscript y in equations (2) to (5). This formulation allows specification of the outer-loop pole constellation in terms of frequency, damping, and real root location for the altitude regulation circuit and for the cross-range regulation circuit. These p-i-d control loops for altitude and cross range are interconnected through the resolver as shown in figure 4. They produce a vertical and lateral acceleration command combination (\ddot{h}_{cmd} , \ddot{y}_{cmd}) that the resolver uses to define a normal load factor and bank angle command combination (n_{cmd} , ϕ_{cmd}).

Interface Between Inner and Outer Loops

The resolver acts as an interface between the inner- and outer-loop control laws described thus far. It couples the altitude and cross-range feedback loops by converting vertical and lateral acceleration command combinations into an aerodynamic lift vector command defined by a normal load factor and bank angle. Consider the coordinate system defined in figure 5, where n_v and n_ℓ are vertical and lateral load factors in g units with respect to an Earth reference frame. (See ref. 11.) For small flight-path angles, n_v and n_ℓ are simply components of the normal load factor n , given by $n_v = n \cos \phi$ and $n_\ell = n \sin \phi$. Point A on figure 5 corresponds to straight and level flight, since the lateral load factor component n_ℓ is zero and the vertical load factor component n_v is equal to that required to maintain constant altitude. Any condition along the horizontal line that passes through point A denotes a level turn, since n_v at any point along this line is equal to the vertical load factor component required to maintain constant altitude. The point designated by B, referred to as the “ballistic origin” throughout this report, denotes the

condition at which the vehicle normal load factor n is zero (zero lift). The vertical distance that separates points A and B is equal to $1 - V^2/Rg$, where V is the vehicle inertial velocity, R is its altitude measured from the Earth’s center, and g is gravity. The term V^2/Rg represents an effect referred to as centrifugal relief. It results from the centripetal acceleration of the vehicle as it flies about a spherical Earth. Lines of constant load factor appear as concentric circles about the ballistic origin as depicted in figure 5. In subsonic flight, the centrifugal relief term is negligible and the horizontal line through point A intersects the vertical axis at a load factor of $1g$ (i.e., a vehicle in straight and level flight experiences $1g$). In hypersonic flight, the relief term becomes appreciable and causes location A to move closer to the ballistic origin, B. Therefore, the aircraft no longer needs to produce a load factor of $1g$ to maintain straight and level flight. At extremely high Mach numbers, the centrifugal relief term can approach $1g$. When $V^2/Rg = 1$, orbital velocity has been achieved, so no aerodynamic lift is required to maintain level flight, and point A coincides with point B in figure 5.

It is possible to define a rectangular-to-polar coordinate transformation that converts desired vertical and lateral accelerations (\ddot{h}_{cmd} , \ddot{y}_{cmd}) into a normal load factor and bank angle command combination (n_{cmd} , ϕ_{cmd}); this combination represents polar coordinates about the ballistic origin. The commanded vertical and lateral accelerations are formulated based on altitude and cross-range errors, as previously described in the outer-loop controls section of this report. Desired load factor components may be expressed in terms of these vertical and lateral acceleration commands as follows:

$$n_{v,\text{cmd}} = \frac{\ddot{h}_{\text{cmd}}}{g} + 1 - \frac{V^2}{Rg} \quad (6a)$$

$$n_{\ell,\text{cmd}} = \frac{\ddot{y}_{\text{cmd}}}{g} \quad (6b)$$

Consider a point defined by a given value of ($n_{v,\text{cmd}}$, $n_{\ell,\text{cmd}}$) as depicted in figure 5. The transformation to polar coordinates is given by

$$n = \sqrt{(n_v)^2 + (n_\ell)^2} \quad (7a)$$

and

$$\phi = \arctan \left(\frac{n_\ell}{n_v} \right) \quad (7b)$$

If the aircraft comes to the angle of attack required to achieve the normal load factor n and rolls to the

bank angle ϕ , then the desired vertical and lateral accelerations (\ddot{h}_{cmd} , \ddot{y}_{cmd}) are produced.

It is apparent from figure 5 and from equations (7a) and (7b) that this coordinate transformation encounters practical difficulties when (n_v, n_ℓ) command combinations lie extremely near the ballistic origin. In this vicinity, lift is relatively small, so large bank angles are required to effect minor changes in cross range. An (n_v, n_ℓ) command sequence that passes directly through the ballistic origin B would produce a discontinuity in commanded bank angle of 180° . For the example design presented subsequently, lower limits on allowable angle-of-attack values specified by engine tolerances result in a minimum allowable load factor and thereby avoid the problems that would be encountered by operating in extremely close proximity to the ballistic origin. In this development, it is assumed that no thrust vectoring is available to augment the lift force and that the only means of achieving a given $(n_{v,\text{cmd}}, n_{\ell,\text{cmd}})$ combination is by modulating angle of attack to produce the desired normal load factor n while rolling the vehicle through bank angle ϕ to redirect the lift vector.

Consideration of Flight Condition Constraints

Not all combinations of $(n_{v,\text{cmd}}, n_{\ell,\text{cmd}})$ produce physically achievable or practical values of n and ϕ . It is therefore necessary to consider boundaries that specify a region of allowable $(n_{v,\text{cmd}}, n_{\ell,\text{cmd}})$ combinations in the resolver coordinate space. Current hypersonic propulsion system concepts are sensitive to variations in angle of attack, angle of sideslip, and dynamic pressure. Recent studies indicate that dramatic variations in thrust can result from angle-of-attack perturbations as large as 2° (ref. 10). For this reason, it is assumed that propulsion system tolerances result in an upper and lower limit on angle of attack about some nominal operating condition. For a given dynamic pressure, the constraints on angle of attack result in normal load factor limits, n_{max} and n_{min} , that would appear as concentric circles about the ballistic origin in figure 5. These limits are shown in figure 6, which portrays one quadrant of the coordinate system from figure 5. The use of these load factor limits to define a region of allowable $(n_{v,\text{cmd}}, n_{\ell,\text{cmd}})$ combinations in the resolver coordinate transformation assures that commanded angles of attack never violate the constraints, regardless of the magnitude of the trajectory error signal.

If an (n_v, n_ℓ) command combination lies outside the boundary prescribed by n_{max} in figure 6, it is possible to determine the lift and bank angle that correspond to this input and then simply limit the

load factor command to n_{max} . This limiting scheme would result in a radial mapping of all points outside the allowable region onto the boundary of a circle of radius n_{max} about the ballistic origin. However, this type of limiting scheme produces some undesirable effects. For example, a pure cross-range command that violates the load factor constraints is represented by point 1 in figure 6. (Any point along the dotted line in fig. 6 corresponds to a level-turn command, since n_v is equal to that required to maintain constant altitude.) If the load factor is simply limited to n_{max} while keeping the bank angle constant, the resulting coordinates would be represented by point 2 in figure 6. Since n_v at point 2 is less than that required for level flight, the aircraft would descend. Such an uncommanded loss of altitude is undesirable, particularly if the guidance system is attempting to follow some maximum dynamic pressure boundary. Therefore, it is clear that simply limiting the load factor command issued by the resolver produces an undesirable effect.

To maintain the commanded altitude in the presence of a limited $(n_{v,\text{cmd}}, n_{\ell,\text{cmd}})$ value, the points outside the allowable region are mapped horizontally onto the boundary instead of radially. This mapping can be achieved by solving for a maximum allowable lateral component of load factor $n_{\ell,\text{max}}$ in terms of the commanded vertical component of load factor $n_{v,\text{cmd}}$. If the point lies outside the boundary and $n_{v,\text{cmd}} < n_{\text{max}}$, then $n_{\ell,\text{max}}$ is obtained by solving the equation for a circle of radius n_{max} about the ballistic origin B as follows:

$$n_{\ell,\text{max}} = \sqrt{n_{\text{max}}^2 - n_{v,\text{cmd}}^2} \quad (8a)$$

A pure cross-range command that violates the load factor constraints is represented by point 1 in figure 7. By using equation (8a), this command is mapped onto the location designated by point 2. It is clear that no altitude loss would result from the use of this limiting scheme. If the point lies outside the boundary, and $n_{v,\text{cmd}} > n_{\text{max}}$ (point 3 in fig. 7), then the command is mapped onto the location designated by point 4. Therefore, the resolver brings the vertical acceleration command to a value less than n_{max} before pursuing the lateral error. In this way, priority has been assigned to the altitude error over the cross-range error. Giving priority to altitude regulation reduces the amount of activity required to compensate for the effect of density variation with altitude on dynamic pressure. This limiting strategy is referred to herein as “altitude priority.”

The preceding discussion provides for the inclusion of an upper load factor limit n_{max} , which results

from the maximum angle-of-attack constraint as a result of propulsion system tolerances. However, the lower limit on angle of attack also implies the existence of a minimum allowable load factor n_{\min} . Some (n_v, n_ℓ) command combinations lie within the radius prescribed by the minimum load factor, such as the descent command depicted by point 5 in figure 7. To achieve the commanded descent rate without violating the minimum load factor constraint, the resolver invokes a strategy whereby the vehicle is banked to attain the desired vertical component of load factor $n_{v,\text{cmd}}$. Therefore, when the command lies within the radius prescribed by n_{\min} , the vertical command is pursued at the expense of incurring a transient cross-range error. This practice is referred to as a “bank-to-dive” strategy throughout this report. The minimum lateral component of load factor $n_{\ell,\min}$ required to enable the descent is defined by the following expression that is analogous to equation (8a):

$$n_{\ell,\min} = \sqrt{(n_{\min})^2 - (n_{v,\text{cmd}})^2} \quad (8b)$$

By using equation (8b), the pure descent command designated by point 5 in figure 7 is mapped onto the bank angle and load factor combination designated by point 6. The bank-to-dive strategy is consistent with the concept of altitude priority. The use of this strategy results in descending “S-turns” in response to large descent commands. A set of conditional statements used to implement this strategy, along with a sketch of their corresponding regions in the resolver coordinate system, is presented in appendix B.

The resolver mapping scheme described above and depicted in figure 7 transforms vertical and lateral acceleration commands from the outer loops into a normal load factor and bank angle command combination to drive the inner loops. During the transformation, load factor limits, altitude-priority, and bank-to-dive features are incorporated.

Throttle Control Law

Hypersonic vehicles may follow trajectories that are defined along a flight condition boundary of constant dynamic pressure. Since high dynamic pressures are generally desirable from the standpoint of propulsive efficiency, and low dynamic pressures are more desirable from the standpoint of aerodynamic heating and thermal management, the trajectory will probably follow a dynamic pressure boundary that represents a compromise between these two factors. It may also be desirable to maintain constant dynamic pressure during hypersonic maneuvers about

the nominal path. For this reason, a throttle controller was designed to cancel out the effect of density and drag changes on dynamic pressure while the vehicle executes maneuvers commanded by the guidance system. This controller is derived from an expression for dynamic pressure rate that includes the effect of density variation with altitude as follows:

$$\dot{q} = \rho_0 V_0 \dot{V} + \frac{V_0^2}{2} \dot{\rho} = \frac{\rho_0 V_0}{m} (T - D) + \frac{V_0^2}{2} C_\rho \dot{h} \quad (9)$$

The throttle controller uses measurements from a longitudinal accelerometer and includes estimates for drag and dynamic pressure error. No dynamic pressure sensor is used. Instead, the dynamic pressure error is obtained by subtracting an estimate based on velocity and altitude from the nominal dynamic pressure. A derivation of the throttle control law is provided in appendix C.

Hypersonic propulsion systems are likely to be sensitive to excessive variation about the nominal operating condition, so large transients in the throttle command signal are undesirable. For this reason, limits are placed on magnitudes of the thrust perturbations commanded by the throttle control law to regulate dynamic pressure.

Implications of Thrust and Angle-of-Attack Limits

Reference has been made to the existence of limits on the allowable angle-of-attack and throttle perturbations. The magnitudes of such limits have a significant impact on the maneuver envelope of a hypersonic vehicle. It is possible to develop expressions for maximum climb rate, descent rate, and turn rate in terms of these limits. In the following paragraphs, these maneuver-rate expressions are presented and their implications are discussed.

First, the limits T_{\max} , T_{\min} , α_{\max} , and α_{\min} are defined as

$$\frac{T_{\max}}{W} = \frac{T_0}{W} + \Delta \left(\frac{T}{W} \right)_{\text{upper limit}} \quad (10a)$$

$$\frac{T_{\min}}{W} = \frac{T_0}{W} - \Delta \left(\frac{T}{W} \right)_{\text{lower limit}} \quad (10b)$$

$$\alpha_{\max} = \alpha_0 + \Delta \alpha_{\text{upper limit}} \quad (11a)$$

$$\alpha_{\min} = \alpha_0 - \Delta \alpha_{\text{lower limit}} \quad (11b)$$

The limits on angle of attack imply upper and lower bounds on load factor, n_{\max} and n_{\min} , which can be expressed as functions of the aerodynamic

model of the vehicle. Assuming a linear lift curve, the load factor limits are

$$n_{\max} = \frac{\bar{q}S}{W} (C_{L,0} + C_{L,\alpha} \alpha_{\max}) \quad (12a)$$

$$n_{\min} = \frac{\bar{q}S}{W} (C_{L,0} + C_{L,\alpha} \alpha_{\min}) \quad (12b)$$

The existence of T_{\max} , together with the desire to maintain constant dynamic pressure, suggests an upper limit on allowable climb rate commanded during excursions from cruise flight conditions. There is some climb rate beyond which the vehicle cannot accelerate rapidly enough to compensate for the effect of density lapse with altitude to maintain dynamic pressure. By determining this maximum climb rate and then applying it to the altitude command input as a rate limit, climb-rate commands that cause violation of the dynamic pressure constraint are avoided. Along a constant dynamic pressure ascent (or descent), climb rate is related to thrust and drag by solving the dynamic pressure rate (eq. (9)) for \dot{h} as follows:

$$\dot{q} = \frac{\rho_0 V_0}{m} (T - D) + \frac{V_0^2}{2} C_\rho \dot{h} = 0 \quad (13)$$

$$\dot{h} = \frac{-2\rho_0}{mV_0 C_\rho} (T - D) \quad (14)$$

Since this climb-rate expression is based on the ability of the vehicle to accelerate or decelerate, drag appears in the expression. To determine the maximum and minimum climb rates that can be accommodated, it is necessary to develop expressions for the maximum and minimum anticipated drag, D_{\max} and D_{\min} , based on the vehicle aerodynamics and angle-of-attack limits. Assuming a parabolic drag polar, the maximum and minimum anticipated drag can be approximated as

$$D_{\max} = \bar{q}S (C_{D,0} + C_{D,\alpha} \alpha_{\max} + C_{D,\alpha^2} \alpha_{\max}^2) \quad (15a)$$

$$D_{\min} = \bar{q}S (C_{D,0} + C_{D,\alpha} \alpha_{\min} + C_{D,\alpha^2} \alpha_{\min}^2) \quad (15b)$$

The positive climb-rate command limit is set for the condition at which the throttle is at maximum in an attempt to compensate for the effect of decreasing density on dynamic pressure, while the resolver is calling for maximum load factor, which results in the maximum angle of attack and maximum drag as follows:

$$\ddot{h}_{\max} = \frac{-2\rho_0}{mV_0 C_\rho} (T_{\max} - D_{\max}) \quad (16a)$$

This limit is somewhat conservative, since greater climb rates could be commanded without violating the dynamic pressure constraint when drag is not at the maximum value. Similarly, the negative climb-rate command limit (maximum descent rate) is set for the condition at which the throttle is at minimum in an attempt to compensate for the effect of increasing density on dynamic pressure, while the resolver is calling for minimum load factor, which results in the minimum angle of attack and minimum drag as follows:

$$\ddot{h}_{\min} = \frac{-2\rho_0}{mV_0 C_\rho} (T_{\min} - D_{\min}) \quad (16b)$$

This limit is also conservative, since greater descent rates could be commanded without violating the dynamic pressure constraint when drag is not at the minimum value. Additionally, the load factor limit implies an upper limit on the turn rate that can be achieved. To avoid commands that are in excess of this maximum achievable turn rate, a rate limit is applied to the heading command input channel. The maximum achievable turn rate at a given flight condition is designated as $\dot{\psi}_{\max}$. The turn-rate equation in terms of load factor, bank angle, and velocity is

$$\dot{\psi} = \frac{ng}{V} \sin \phi \quad (17)$$

The maximum turn-rate command limit is set at the maximum load factor and maximum allowable bank angle as follows:

$$\dot{\psi}_{\max} = \frac{n_{\max}g}{V} \sin \phi_{\max} \quad (18)$$

This equation expresses a rate command limit that is applied to changes in commanded heading angle. The maximum allowable bank angle used in equation (18) is depicted in figure 8. At larger bank angles, it would no longer be possible to achieve \ddot{h}_{cmd} without exceeding the load factor limit. From the figure, it is clear that the maximum allowable bank angle for use in equation (18) is given by

$$\phi_{\max} = \cos^{-1} \left(\frac{1 - \frac{V^2}{Rg} + \ddot{h}_{\text{cmd}}}{n_{\max}} \right) \quad (19)$$

In the control system described in this report, the limits on climb-rate and turn-rate commands are implemented as shaping blocks in the input signal paths for altitude and heading immediately following the guidance system block in figure 2. In fact, it is probable that the guidance system itself would apply

such limits rather than simply issuing step commands in altitude, cross range, or heading.

The expressions presented in this section illustrate the manner in which limits on allowable angle-of-attack and throttle perturbations, together with the desire to maintain constant dynamic pressure, influence the maneuver envelope of a hypersonic vehicle about a steady cruise flight condition. Some maneuvers, however, may represent transitions between steady cruise flight conditions. In such cases, the nominal throttle setting and angle of attack might be allowed to vary during the maneuver. However, similar limiting expressions may still apply based on the levels of perturbations that could be tolerated about some smoothly varying nominal. The turn-rate and climb-rate limits would then express the acceptable levels of departure from the transition flight path.

Design Example and Test Simulation

To test the control concept presented in this paper, an unpiloted simulation was developed by using a representative hypersonic single-stage-to-orbit configuration as an example vehicle. A sketch of this hypersonic winged-cone configuration is presented in figure 9. The fuselage has a 5° conical forebody with wraparound engine nacelles and a 75° swept delta wing. The configuration includes a rudder and elevons for vertical and lateral control. Table I is a summary of geometric characteristics for this configuration. A considerable aerodynamic data base has been generated for this configuration as a result of extensive wind-tunnel testing and analytical investigations with the aerodynamic preliminary analysis system (APAS) of reference 12. The geometric, propulsive, and aerodynamic data for this configuration are detailed in references 13 and 14. Aerodynamic data are included in tables II and III. The inner-loop, outer-loop, and resolver elements were designed by using this aerodynamic and propulsion model for the example vehicle.

For the purposes of this study, the configuration was trimmed at four different hypersonic flight conditions along a 2000 lb/ft^2 dynamic pressure trajectory generated by the program to optimize simulated trajectories (POST) discussed in reference 15. Conditions were selected at Mach numbers of 6, 10, 15, and 20. Trim values of angle of attack, velocity, and altitude are presented in table IV for these four flight conditions. Inertial quantities and trim parameters that correspond to these four nominal cruise conditions were used in the feedback gain and limit expressions for all control system elements presented previously in this report and in appendix A. Frequency and damping parameters used in the design example,

and the equation numbers to which they apply, are presented in table V. The control design can be similarly adapted to any desired configuration by substituting the appropriate vehicle constants into these expressions and selecting the desired frequency and damping parameters.

To implement the climb-rate and turn-rate limits presented previously, it was necessary to estimate the magnitude of allowable thrust and angle-of-attack perturbations about the trim settings. For the example design, angle-of-attack perturbations about the trim value commanded to perform the given maneuvers were limited to $\pm 0.4^\circ$. The magnitudes of such limits for an actual working vehicle are a topic of current research, and these values may or may not be overly restrictive. However, one objective of this study is to investigate the manner in which propulsion and flight condition constraints impact the maneuvering capabilities of hypersonic aerospacecraft. These effects may be investigated without exact knowledge of the constraint magnitudes, so this simulation has been conducted in the interest of revealing some issues of importance to this general class of vehicles. For this design example, throttle limits were placed at -0.10 and 0.30 about the trim thrust-to-weight ratio for straight and level cruise. According to the propulsive data base presented in reference 13, these variations are well within the range achievable by varying the fuel equivalence ratio for a representative vehicle. Therefore, the limit quantities on thrust and angle-of-attack perturbations for this example vehicle are defined as

$$\frac{T_{\max}}{W} = \frac{T_0}{W} + 0.30 \quad (20a)$$

$$\frac{T_{\min}}{W} = \frac{T_0}{W} - 0.10 \quad (20b)$$

$$\alpha_{\max} = \alpha_0 + 0.4^\circ \quad (21a)$$

$$\alpha_{\min} = \alpha_0 - 0.4^\circ \quad (21b)$$

An accurate model for the dynamics of a propulsion system to be used on a hypersonic air-breathing vehicle was not available, but very rapid response was assumed possible for this type of engine. For this example, a perfect engine model was assumed in which the commanded thrust was equal to the achieved thrust. Also, the model did not include thrust variations with angle of attack.

A test simulation of the resolver control system and the example vehicle was constructed by using a control design and analysis software tool that allows simulation of all the nonlinear elements described in

this report. In this batch simulation, the resolver was used to drive the inner-loop controls directly, instead of driving a cockpit flight director as would be the case in a piloted simulation; also, perfect angle-of-attack control was assumed, in which the commanded angle of attack was equal to the achieved angle of attack (short-period dynamics were omitted). Time histories were produced to test the response of the controlled vehicle to various guidance commands and atmospheric disturbances. In this way, the ability of the controller to complete a specified guidance task was evaluated.

A series of example guidance commands were generated to simulate the execution of several maneuvering tasks. Guidance inputs to the controller consisted of vertical offset commands, lateral offset commands, and heading changes. Responses to several command combinations, as well as a maneuver that utilized the bank-to-dive feature of the resolver, were evaluated. Responses to three forms of atmospheric disturbances were also investigated. The hypersonic simulation was subjected to headwind gusts, crosswind gusts, and density variations. It is important to examine the vehicle response to such atmospheric disturbances, since the disturbances may give rise to angle-of-attack and angle-of-sideslip excursions, which are a major concern in designing controllers for hypersonic vehicles. All time histories presented in this report represent variations about the trim values shown in table IV. Table VI is a summary of the response plots in terms of figure number, input type, and flight condition. Response time histories from the batch simulation are discussed in the following sections. These sections are intended to examine the trends and effects apparent in the response time histories at Mach numbers of 6, 10, 15, and 20, and to relate these effects back to their physical causes.

Responses to Guidance Commands

Simulated perturbation responses to an altitude step command of 2000 ft for Mach numbers of 6, 10, 15, and 20 are shown in figure 10. The altitude responses reveal a large variation in the times required to achieve the commanded altitude at the different flight conditions. The length of time required to achieve the altitude command is primarily a function of the climb-rate limits presented in equations (16). The variation in speed of response at the four Mach numbers is mainly caused by the differing values of maximum thrust (T_{\max}) at each flight condition and is therefore a result of the manner in which limits were implemented in this specific example design. As a result of observing the climb-rate

command limit, dynamic pressure variations during the maneuver were maintained to $\pm 2 \text{ lb/ft}^2$ about the nominal 2000 lb/ft^2 condition. The angle-of-attack responses presented in figure 10 indicate an initial increase above the trim value to initiate the climb, followed by a marked decrease to a new trim value at the higher altitude. This trim angle-of-attack decrease in response to the altitude change along a constant dynamic pressure trajectory was caused by two factors of particular significance in hypersonic flight. First, the density lapse with altitude required the vehicle to accelerate to maintain constant dynamic pressure; this effect was apparent in the velocity time histories of figure 10. Second, it was stated previously that the effect of centrifugal relief in hypersonic flight is to reduce the load factor required to maintain constant altitude as velocity increases (eq. (1)) as follows:

$$n = 1 - \frac{V^2}{Rg}$$

Since the vehicle increased velocity to maintain dynamic pressure at the higher altitude, centrifugal relief was greater at the new trim condition. Therefore, the new trim condition required less lift and a lower angle of attack to maintain straight and level flight.

The throttle variations required to regulate dynamic pressure during the maneuver are presented in terms of thrust-to-weight ratio in figure 10. The Mach 15 and Mach 20 cases reached the maximum thrust-perturbation limit during the ascent. Trim throttle settings changed slightly in response to the lower angle of attack at the end of the maneuver.

Perturbation responses to a cross-range step command of 20 000 ft are shown in figure 11. The cross-range time histories are identical for all Mach numbers, since the outer loop was designed to the same frequency and damping in each case. Although cross-range time histories are identical for the four flight conditions, their corresponding ground tracks differ dramatically since the trim velocities are dissimilar. (See table IV.) The responses in figure 11 show that much larger bank angles were required to produce a given lateral acceleration at higher velocities; these responses illustrate the influence of centrifugal relief on high-speed turns. The heading responses in figure 11 show that, as expected, a given lateral acceleration produced a larger change in heading at lower velocities. As the β response indicates, the inner-loop controls were designed to keep sideslip to a minimum during lateral maneuvers. Altitude variations were kept to within 10 ft of the trim setting, and angle-of-attack variations required to produce the desired

load factors commanded by the resolver did not reach the 0.4° limit. Small thrust and velocity variations were produced by the throttle control law as it acted to regulate dynamic pressure during the maneuver.

A polar plot of load factor versus bank angle in the resolver coordinate system for the 20 000-ft cross-range offset maneuver is shown in figure 12. It is clear from figure 5 that the time histories in figure 12 represent excursions of bank angle and load factor commands along the horizontal line for level turns at each of the four flight conditions. The dramatic reduction in vertical load factor required to maintain level flight at the higher Mach numbers is apparent. This reduction in vertical load factor as a result of centrifugal relief causes an increase in bank angle required to produce a given lateral acceleration in level flight. (See fig. 11.)

Responses to a vertical command of 2000 ft combined with a lateral offset command of 20 000 ft are shown in figure 13. The resolver produced what is essentially a superposition of the responses from figures 10 and 11. Dynamic pressure was regulated to within $\pm 2 \text{ lb/ft}^2$ of nominal during this maneuver. All the trends that were apparent in figures 10 and 11 as a result of the flight condition constraints and centrifugal relief effects are also apparent in these responses.

Commanded changes in heading angle were represented by ramp inputs in the cross-range command signal path. The maximum turn-rate command limit presented in equation (18) was applied to this input signal. Responses to a commanded heading change of 10° are shown in figure 14. The variation in speed of response is a result of the effect of different trim velocities on the maximum turn-rate command in equation (18). Exceeding this turn-rate command limit would have resulted in an uncommanded loss of altitude in response to the 10° heading change. Again, the bank angle required to produce a given lateral acceleration increased with Mach number. Altitude perturbations were kept to a minimum, and angle-of-attack variations reached the upper limit of 0.4° for those flight conditions that required sustained operation at the maximum allowable turn rate. Clearly, at the higher Mach numbers, even minor heading changes can require large amounts of time. The throttle controller produced minor thrust variations, which compensated for the higher drag at larger angles of attack, to regulate dynamic pressure throughout this heading-change maneuver.

The concept of a bank-to-dive maneuver was described previously in this report. A bank-to-dive maneuver allows the vehicle to remain at or above the

minimum allowable angle of attack while achieving a high rate of descent. By rolling the lift vector off of vertical and maintaining the minimum allowable angle of attack, a faster rate of descent is realized than would otherwise be possible. Because of the significant effect of density variation with altitude, it was necessary to relax the constraint of maintaining constant dynamic pressure to execute a bank-to-dive maneuver for the hypersonic vehicle used in this example. Since density increases as the vehicle descends, it is necessary to reduce speed to maintain constant dynamic pressure. The maximum allowable descent-rate command presented in equation (16b) was derived to prevent descent commands that would require the vehicle to decelerate faster than possible. However, to enter the region of figure 7 where the commanded load factor was less than n_{\min} , it was necessary to allow a descent-rate command in excess of the value expressed in equation (16b). Time histories for a descent command step of 5000 ft at Mach 15 with the bank-to-dive feature are shown in figure 15. This response was compared with a response to the same descent command without relaxation of the dynamic pressure constraint and consequential descent-rate limit. When the descent-rate command limit was not relaxed, the bank-to-dive feature of the resolver was never invoked, so a pure vertical descent resulted (no lateral activity). It is apparent from figure 15 that the bank-to-dive maneuver produces a much faster response at the expense of incurring a transient cross-range excursion and dynamic pressure error. The vehicle was unable to decelerate rapidly enough to remain on the dynamic pressure boundary during the bank-to-dive descent. The cross-range and bank angle excursions were plotted on a time scale of 200 sec rather than 800 sec, since no lateral activity occurred for the remainder of the maneuver. The lateral flight-path deviation remained within $\pm 3000 \text{ ft}$, and the bank angle excursion was about $\pm 50^\circ$. The trim angle of attack at the end of the maneuver was higher than at the start of the maneuver. The deceleration reduced the centrifugal relief term, V^2/Rg , so the vehicle had to produce more lift to remain in level flight. (The dynamic pressure was the same at the beginning and end of the maneuver, so it is not a matter of requiring a greater angle of attack to produce the same amount of lift.)

A comparison of the bank-to-dive responses to the 5000-ft descent command for Mach 6, 10, 15, and 20 is shown in figure 16. Although the altitude perturbations are almost identical, the dynamic pressure excursions are greater at the faster and higher flight conditions. It is possible that the addition of a deployable drag-producing device would enable

the vehicle to decelerate rapidly enough to utilize a bank-to-dive maneuver scheme while satisfying the constant dynamic pressure constraint, although the need for such a device would be configuration and flight condition dependent. Larger bank angle excursions were again experienced at the faster flight conditions as a result of greater centrifugal relief. The bank-to-dive method might be used during reentry, abort, or nonorbital missions.

Responses to Atmospheric Disturbances

Responses to three types of atmospheric disturbances were simulated at each of the four hypersonic flight conditions. The first type of disturbance was a 30-knot headwind with a 10-mile ramp onset. The second type of disturbance was a 30-knot crosswind with a 10-mile ramp onset. After reaching a peak value of 30 knots, the headwind or crosswind remained constant through the rest of the simulation. The third type of disturbance was an atmospheric-density perturbation represented as a 10-percent decrease below nominal in the form of one cycle of a negative ($1 - \cos$) wave. This disturbance was intended to represent flight through a region of rarefied air in the upper atmosphere. Such density variations have been recorded during shuttle reentry flights at very high altitudes (ref. 16), although much uncertainty still exists as to the structure and magnitude of density perturbations that a hypersonic aerospacecraft might encounter. The negative 10-percent density pulse was defined with a horizontal length of 10 miles. Since these disturbances were defined spatially (in terms of miles), the vehicle experienced the perturbations at different rates in the time domain, depending on trim velocity of the flight condition. Recent studies indicate that the severity of these disturbances is light compared with the strength of perturbations that may be encountered in the actual environment (ref. 16). Therefore, these responses are used to illustrate the general manner in which a hypersonic vehicle equipped with the control design presented in this report would react to such disturbances; these responses should not be interpreted as an exhaustive study of robustness to environmental perturbations.

Responses to the 30-knot headwind with a 10-mile ramp onset at each of the four flight conditions are shown in figure 17. Very small angle-of-attack and altitude variations were experienced. Dynamic pressure and thrust variations were plotted on a shorter time scale, since these transients occurred more rapidly. The throttle control law reduced the thrust level to the lower limit to decelerate the vehicle as rapidly as possible; this deceleration compensated

for the effect of the headwind on dynamic pressure. As previously mentioned, a model of the engine dynamics for the hypersonic propulsion system was not readily available for use in this simulation. The throttle transients in response to the headwind were quite rapid, and an accurate model of the engine dynamics might influence this response and possibly require some modification or tuning of the throttle control law.

Cross-range deviation and sideslip responses to the 30-knot crosswind with a 10-mile ramp onset are shown in figure 18. Perturbations in sideslip angle were less than a tenth of a degree, and there were negligible changes in angle of attack, thrust, bank angle, and heading. The resulting cross-range deviations were negligible, although they took longer to eliminate. (Note the longer time scale on cross range.) At hypersonic speeds, the longitudinal component of velocity greatly diminished the effect of any crosswind component.

Responses to a negative 10-percent density pulse with a horizontal length of 10 miles are shown in figure 19. Transients that resulted from the density pulse were greatest at the slowest flight condition. Angle-of-attack and throttle variations remained within the perturbation limits specified previously in this report for the design example. The effect on dynamic pressure was plotted on a time scale of 20 sec, since the vehicle traversed the disturbance so rapidly. It is clear that the throttle controller could not act quickly enough to neutralize the dynamic pressure excursion produced by such a disturbance.

The spatial dimension of the density pulse was varied at the Mach 10 flight condition to examine the effect of disturbance duration on the vehicle response. Responses at the Mach 10 flight condition to a negative 10-percent density pulse with lengths of 5, 10, 15, and 20 miles are compared in figure 20. The longer pulses produced the greatest transients, although perturbations again remained within the limits specified previously in this report. The throttle control was again unable to respond with sufficient speed to neutralize the effect of density variations on dynamic pressure. In the resolver control system, altitude is regulated as a trajectory variable, while dynamic pressure is handled by an inner-loop throttle control law. This type of control scheme is well-suited to regulation of dynamic pressure during a planned maneuver, but it cannot provide perfect regulation of dynamic pressure in response to atmospheric density perturbations. To completely cancel the effect of density variations on dynamic pressure, the vehicle would be required to change velocity or

altitude with a faster response than appears to be reasonable. The inability to completely compensate for the effects of density variations on dynamic pressure suggests that a practical hypersonic propulsion system must be able to tolerate a certain level of dynamic pressure fluctuation. This finding is consistent with previous studies that examined various implications of regulating different trajectory parameter combinations. (See refs. 2 to 6.)

Concluding Remarks

A control design option is presented for executing coordinated maneuvers in hypersonic flight while regulating key parameters such as angle of attack, angle of sideslip, and dynamic pressure. The design involves the use of a coordinate transformation, referred to as a “resolver,” which converts vertical and lateral acceleration commands into an aerodynamic lift vector command specified by a normal load factor and bank angle. The vertical and lateral acceleration commands are based on feedback errors from commanded values of altitude and cross range. The control system was applied to an example configuration and implemented in an unpiloted digital simulation at Mach numbers of 6, 10, 15, and 20. The simulation was used to illustrate the manner in which the control system responds to various commands and atmospheric disturbances.

A vehicle in hypersonic flight may approach orbital velocities while still in the atmosphere. The centripetal acceleration of the vehicle as it circles the Earth then becomes significant enough to cause a noticeable reduction in the load factor required to maintain constant altitude. This effect, referred to as centrifugal relief, was demonstrated in the responses of the simulation to commanded altitude, heading, and cross-range changes. A dramatic reduction in vertical load factor required to maintain level flight at the higher Mach numbers was observed. This reduction in vertical load factor caused the bank angle that was required to produce a given lateral acceleration to increase with Mach number. Centrifugal relief also caused changes in trim angle of attack in response to altitude variations along a constant dynamic pressure trajectory. The control design presented in this report accounts for the effect of centrifugal relief while tracking altitude and cross-range commands. It also incorporates load factor limits, prioritizes altitude regulation over cross-range regulation, and includes a feature whereby the lift vec-

tor can be rolled off of vertical to achieve a desired descent rate at the expense of incurring a transient cross-range error (“bank-to-dive”).

Several implications of propulsion and flight condition constraints for the maneuvering of air-breathing hypersonic vehicles were examined. Angle-of-attack and throttle perturbation constraints, combined with centrifugal relief effects and the desire to maintain constant dynamic pressure, significantly impact the maneuver envelope for such vehicles. Turn-rate, climb-rate, and descent-rate limits with respect to maneuvering about some hypersonic cruise condition were expressed in terms of these constraints. Density variation with altitude strongly influences climb-rate and descent-rate limits and throttle modulation if dynamic pressure is to be maintained during vertical transitions between cruise flight conditions. In particular, the high descent rates that are achievable by using a bank-to-dive maneuver scheme required a departure from the dynamic pressure constraint, since the vehicle was unable to decelerate rapidly enough to compensate for the effect of density lapse on dynamic pressure. However, the bank-to-dive method might be used during reentry, abort, or nonorbital missions.

Of the atmospheric disturbances investigated, density pulses caused the greatest transient response. The findings of this study indicate that hypersonic propulsion systems should be designed with the capacity to tolerate fluctuations in dynamic pressure that result from high-speed flight through atmospheric density perturbations. The effects of atmospheric disturbances on advanced propulsion systems must be investigated and provided for in any practical hypersonic design.

This research has defined an automatic control design for executing coordinated maneuvers in hypersonic flight while regulating key flight condition parameters. The primary element of this control design provides an environment for the incorporation of various constraints associated with high-speed flight. Additionally, this work has illustrated the manner in which propulsion and flight condition constraints impact the maneuvering capabilities of air-breathing hypersonic aerospacecraft.

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Appendix A

Development of Inner-Loop Flight Controls

This appendix describes the derivation of the lateral inner loops, which were designed to stabilize the vehicle dynamics, track bank angle commands from the resolver, and maintain zero sideslip during the maneuver. A simple pole-placement technique is used that allows specification of the inner-loop responses in terms of desired frequency and damping. Two separate control loops are synthesized to drive the rudder and the ailerons. The estimation of sideslip angle with a lateral accelerometer and the concept of a complementary filter to reduce high-frequency control action that results from atmospheric disturbances are also described. Although a relatively simple inner-loop design technique has been used for this investigation, more sophisticated designs can also be used in conjunction with the resolver concept presented in this report.

Rudder Control Law

The rudder is used to maintain zero sideslip during a maneuver. First, a Dutch Roll approximation is applied to address the yaw control, and only the yaw-rate and sideslip equations are given as follows:

$$\dot{r} = N_\beta \beta + N_p p + N_r r + N_{\delta a} \delta_a + N_{\delta r} \delta_r + \frac{I_{xz}}{I_z} \dot{p} \quad (\text{A1})$$

$$\dot{\beta} = Y_\beta \beta + Y_p p + Y_r r + Y_{\delta a} \delta_a + Y_{\delta r} \delta_r - r + \frac{g}{V} \cos \theta_0 \sin \phi \quad (\text{A2})$$

The final term in equation (A2) involves gravity and is negligible in hypersonic flight since velocity is very large. Also, the effect of rotary derivatives is often small at hypersonic speeds, since velocity appears in the denominator of the dimensionalizing expression. If the rotary derivative and inertial coupling terms in equations (A1) and (A2) are neglected, a Laplace transformation yields the following dynamic system:

$$sr(s) = N_\beta \beta(s) + N_{\delta a} \delta_a(s) + N_{\delta r} \delta_r(s) \quad (\text{A3})$$

$$s\beta(s) = Y_\beta \beta(s) + Y_{\delta a} \delta_a(s) + Y_{\delta r} \delta_r(s) - r(s) \quad (\text{A4})$$

A rudder deflection command is defined using yaw rate and angle-of-sideslip feedback of the following form:

$$\delta_r(s) = Kr_\beta \beta(s) + Kr_r r(s) \quad (\text{A5})$$

Substituting the rudder deflection command into expressions (A3) and (A4) and neglecting yaw and side

force due aileron deflection yields

$$sr(s) - N_\beta \beta(s) - N_{\delta r} [Kr_\beta \beta(s) + Kr_r r(s)] = 0 \quad (\text{A6})$$

$$s\beta(s) + r(s) - Y_\beta \beta(s) - Y_{\delta r} [Kr_\beta \beta(s) + Kr_r r(s)] = 0 \quad (\text{A7})$$

This system can be expressed in matrix form as

$$\begin{bmatrix} s - N_{\delta r} Kr_r & -N_\beta - N_{\delta r} Kr_\beta \\ 1 - Y_{\delta r} Kr_r & s - Y_\beta - Y_{\delta r} Kr_\beta \end{bmatrix} \begin{bmatrix} r(s) \\ \beta(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (\text{A8})$$

This matrix yields the following second-order characteristic equation for the closed-loop system:

$$s^2 - (Y_\beta + Y_{\delta r} Kr_\beta + N_{\delta r} Kr_r) s + [N_\beta + N_{\delta r} Kr_\beta + Kr_r (N_{\delta r} Y_\beta - Y_{\delta r} N_\beta)] = 0 \quad (\text{A9})$$

The coefficients of this characteristic equation are expressed below in terms of frequency ω_r and damping ζ_r for the second-order system:

$$2\zeta_r \omega_r = -(Y_\beta + Y_{\delta r} Kr_\beta + N_{\delta r} Kr_r) \quad (\text{A10})$$

$$\omega_r^2 = N_\beta + N_{\delta r} Kr_\beta + Kr_r (N_{\delta r} Y_\beta - Y_{\delta r} N_\beta) \quad (\text{A11})$$

Neglecting the relatively small terms, $Kr_r (N_{\delta r} Y_\beta - Y_{\delta r} N_\beta)$, and solving for the feedback gains, Kr_β and Kr_r , in terms of frequency and damping, yields

$$Kr_\beta = \frac{\omega_r^2 - N_\beta}{N_{\delta r}} \quad (\text{A12a})$$

and

$$Kr_r = \frac{2\zeta_r \omega_r + Y_\beta + Y_{\delta r} Kr_\beta}{-N_{\delta r}} \quad (\text{A12b})$$

It is now possible to quickly determine the feedback gains required to produce a given closed-loop frequency and damping combination. This combination allows the designer to place the closed-loop poles at a location that might be desirable from a handling-qualities perspective.

Angle-of-sideslip estimation. The reliability of air-data measurements during some phases of hypersonic flight may be uncertain. The control system designed in this report utilizes an angle-of-sideslip feedback that is constructed from a lateral-accelerometer reading rather than an air-data sensor array. The lateral accelerometer is placed at the instantaneous center of rotation in response to a rudder

deflection; this placement eliminates the need to correct the accelerometer reading for rudder effects. It would actually also be necessary to filter the signal appropriately to eliminate corruption of the measurement as a result of structural vibrations. The equation for the lateral acceleration measured by an accelerometer located a distance λ forward of the center of gravity is given by

$$A_{Y,acc} = V_0 (Y_\beta \beta + Y_{\delta_r} \delta_r) + \lambda (N_\beta \beta + N_{\delta_r} \delta_r) \quad (A13)$$

Solving this expression for β yields an angle-of-sideslip measurement, in terms of the accelerometer reading and the rudder deflection, of

$$\hat{\beta} = \frac{A_{Y,acc} - \delta_r (V_0 Y_{\delta_r} + \lambda N_{\delta_r})}{V_0 Y_\beta + \lambda N_\beta} \quad (A14)$$

By placing the accelerometer at the lateral center of rotation, it is possible to eliminate the influence of rudder deflection on the angle-of-sideslip measurement. If

$$\lambda = \frac{-V_0 Y_{\delta_r}}{N_{\delta_r}} \quad (A15a)$$

then

$$\hat{\beta} = \frac{A_{Y,acc}}{V_0 Y_\beta + \lambda N_\beta} \quad (A15b)$$

This expression for sideslip can be substituted into equation (A5) to make the lateral inner-loop controller independent of air-data measurements. The expression for λ depends on parameters that change with flight condition; the lateral center of rotation is not constant over the range of hypersonic flight conditions experienced by the vehicle. Since it is unreasonable to relocate the accelerometer as flight condition varies, it is probable that the instrument would be placed at some location that represents the average position of the center of rotation for those flight conditions at which the air-data system measurements are the least reliable. For the simulated time histories presented in this report, which include flight conditions at Mach 6, 10, 15, and 20, the lateral accelerometer was placed at the center of rotation for the Mach 6 flight condition.

Complementary filter. Sudden crosswind gusts are fed directly into the lateral control-surface deflections via sideslip feedback under the current rudder control law. To avoid sharp peaks in the commanded position of control surfaces, a low-pass filter

is added to the sideslip-angle feedback loop. The filter is complemented with derivative feedback to produce a unity transfer function, which eliminates its effect on the stability of the closed-loop system and prevents it from interfering with the response to pilot or guidance system commands. Because of the derivative feedback through the filter, this network is referred to as a complementary filter. The expression for the filtered angle-of-sideslip feedback is

$$\tilde{\beta} = \frac{\tau \hat{\beta}}{\tau s + 1} + \frac{\hat{\beta}}{\tau s + 1} \quad (A16)$$

where $\hat{\beta}$ is obtained from equation (A14) and $\tilde{\beta}$ is given by

$$\hat{\beta} = Y_\beta \hat{\beta} + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r - r \quad (A17)$$

Rotary derivatives have been neglected in equation (A17). The time constant can be selected to provide a reasonable amount of disturbance rejection and a favorable response characteristic. Expression (A16) can be substituted for sideslip feedback in the rudder control law (eq. (A5)). Ideally, the complementary filter has a unity transfer function and does not affect the stability of the closed-loop system. In practice, the degree to which the transfer function of equation (A16) differs from unity depends on the accuracy of the estimated derivative signal used in this equation. To this extent, the complementary filter acts to reduce high-frequency control action in response to atmospheric disturbance inputs but not to commanded control inputs.

Aileron Control Law

The roll-control law drives the ailerons to achieve bank angle commands issued by the resolver. The roll-rate and bank angle equations are as follows:

$$\dot{p} = L_\beta \beta + L_p p + L_r r + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r + \frac{I_{xz}}{I_x} \dot{r} \quad (A18)$$

$$\dot{\phi} = p + r \tan \theta_0 \cos \phi \quad (A19)$$

Neglecting the rotary derivative and inertial coupling terms in equation (A18) yields

$$\dot{p} = L_\beta \beta + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r \quad (A20)$$

The simplifying assumption is made that $\dot{\phi} \approx p$ in equation (A19). A feedback control law for aileron deflection that allows decoupling of yawing

and rolling motions by eliminating the effect of rudder deflection and angle of sideslip on equation (A20) is defined as follows:

$$\delta_a = K a_\beta \beta + K a_{\delta r} \delta_r + K a_\phi \phi_{\text{err}} + K a_{\dot{\phi}} \dot{\phi} \quad (\text{A21})$$

In equation (A21), ϕ_{err} is defined as the commanded roll angle minus the actual roll angle. This control law also includes roll-attitude and roll-rate feedback, which provides a means of specifying frequency and damping in roll response. The angle-of-sideslip signal used in this expression is the complementary filtered estimate in equation (A16) from the lateral accelerometer. The rudder deflection command is also used in this control law. Substituting the aileron control law into equation (A20) yields

$$\ddot{\phi} = L_\beta \beta + L_{\delta r} \delta_r + L_{\delta a} \left(K a_\beta \beta + K a_{\delta r} \delta_r + K a_\phi \phi_{\text{err}} + K a_{\dot{\phi}} \dot{\phi} \right) \quad (\text{A22})$$

To achieve the desired decoupling of roll and yaw responses, let

$$K a_\beta = \frac{-L_\beta}{L_{\delta a}} \quad (\text{A23a})$$

and

$$K a_{\delta r} = \frac{-L_{\delta r}}{L_{\delta a}} \quad (\text{A23b})$$

which, upon substitution into equation (A22), yields

$$\ddot{\phi} = L_{\delta a} K a_\phi \phi_{\text{err}} + L_{\delta a} K a_{\dot{\phi}} \dot{\phi} \quad (\text{A24})$$

A Laplace transform of equation (A24) yields the following second-order characteristic equation:

$$s^2 - L_{\delta a} K a_{\dot{\phi}} s - L_{\delta a} K a_\phi = 0 \quad (\text{A25})$$

The feedback gains in this expression are easily solved for in terms of desired frequency and damping of the closed-loop system to yield

$$K a_\phi = \frac{\omega_a^2}{-L_{\delta a}} \quad (\text{A26a})$$

and

$$K a_{\dot{\phi}} = \frac{2\zeta_a \omega_a}{-L_{\delta a}} \quad (\text{A26b})$$

These gains, along with those in equations (A23), are used in equation (A21) to produce the final aileron control law. The rotary terms neglected in the derivation of this control law were of little significance at the hypersonic flight conditions that were simulated. The inherent aerodynamic roll damping of the vehicle is negligible at these flight conditions, so most of the damping is supplied by the roll-rate feedback loop in the aileron control law.

Appendix B

Conditional Statements Used in Bank-to-Dive Strategy

This appendix describes some conditional statements that are needed to successfully implement the bank-to-dive limits in the resolver coordinate system. Three regions in the resolver coordinate system are illustrated in sketch A. Region 1 consists of all points for which the normal load factor is less than n_{\min} , the minimum allowable load factor prescribed by equation (12b). Region 2 consists of all points outside of region 1 for which $n_{v,\text{cmd}}$ is less than n_{\min} . Region 3 consists of all points outside of regions 1 and 2. Once

a bank-to-dive maneuver has been initiated by a command lying within region 1, a lateral error is incurred to achieve the desired descent rate. It is then necessary to suspend the normal operation of the resolver until the acceleration command no longer lies within the areas designated as regions 1 and 2 in sketch A. Once the acceleration command sequence has passed into region 3, the lateral error can be eliminated. Omission of these conditional statements would result in a bank angle command chatter between left and right halves of the resolver coordinate plane each time the acceleration command sequence passes from region 1 to region 2. In the sketch, B-T-D is a condition that can be either true or false, depending on the location of the current acceleration command.

Sketch A

Appendix C

Development of Throttle Control Law

This appendix describes the derivation of a feedback control law for the throttle that has been designed to regulate dynamic pressure during a maneuver. The throttle control law is derived from the expression for dynamic pressure rate, which is given by

$$\dot{q} = \rho_0 V_0 \dot{V} + \frac{V_0^2}{2} \dot{\rho} \quad (\text{C1a})$$

where velocity variations and density lapse with altitude may be approximated as

$$\dot{V} = \frac{T - D}{m} \quad (\text{C1b})$$

and

$$\dot{\rho} = \frac{\partial \rho}{\partial h} \dot{h} = C_\rho \dot{h} \quad (\text{C1c})$$

The term C_ρ is the density lapse coefficient for a given flight condition. Substitution of equations (C1b) and (C1c) into equation (C1a) yields

$$\dot{q} = \frac{\rho_0 V_0}{m} T - \frac{\rho_0 V_0}{m} D + \frac{V_0^2}{2} C_\rho \dot{h} \quad (\text{C2})$$

A throttle control law is defined, based on equation (C2), to compensate for the effect of altitude and drag variations while providing first-order regulation of dynamic pressure as follows:

$$T_{\text{cmd}} = D - \frac{m V_0 C_\rho}{2 \rho_0} \dot{h} + K_{\bar{q}} \bar{q}_{\text{err}} \quad (\text{C3})$$

Substitution of T_{cmd} for T in equation (C2) produces

$$\dot{q} = \frac{\rho_0 V_0}{m} K_{\bar{q}} \bar{q}_{\text{err}} \quad (\text{C4})$$

and

$$\bar{q}(s) \left(s - \frac{\rho_0 V_0}{m} K_{\bar{q}} \right) = 0 \quad (\text{C5})$$

which yields the following first-order characteristic equation:

$$s + \sigma_{\bar{q}} = 0 \quad (\text{C6a})$$

where

$$\sigma_{\bar{q}} = \frac{-\rho_0 V_0}{m} K_{\bar{q}} \quad (\text{C6b})$$

The dynamic pressure feedback gain can be expressed in terms of the real root in the following first-order characteristic:

$$K_{\bar{q}} = \frac{-m \sigma_{\bar{q}}}{\rho_0 V_0} \quad (\text{C7})$$

The throttle controller described in equation (C3) includes a drag estimate and a dynamic pressure error feedback. Drag is estimated by solving equation (C1b) as follows:

$$\hat{D} = T_{\text{prev}} - m \dot{V}_{\text{acc}} \quad (\text{C8})$$

where the inertial velocity variation is obtained from a longitudinal accelerometer and T_{prev} represents the thrust command from the previous time step. The dynamic pressure error is obtained by subtracting an estimate based on velocity and altitude from the nominal dynamic pressure. Therefore,

$$\bar{q}_{\text{err}} = \bar{q}_0 - \hat{\bar{q}} \quad (\text{C9})$$

where the dynamic pressure estimate is given by

$$\hat{\bar{q}} = \frac{\hat{\rho} V^2}{2} = \frac{V^2}{2} [\rho_0 + C_\rho (h - h_0)] \quad (\text{C10})$$

Substituting the drag and dynamic pressure estimates from equations (C8), (C9), and (C10) into equation (C3) yields the final throttle control law as follows:

$$T_{\text{cmd}} = T_{\text{prev}} - m \dot{V}_{\text{acc}} - \frac{m V_0 C_\rho}{2 \rho_0} \dot{h} + K_{\bar{q}} \times \left\{ \bar{q}_0 - \frac{V^2}{2} [\rho_0 + C_\rho (h - h_0)] \right\} \quad (\text{C11})$$

The real root in equations (C6) should be chosen to prevent excessive dynamic pressure excursions without producing extreme throttle transients. This development of the throttle control law depends heavily on an accurate model of atmospheric density variation with altitude, and its performance may be strongly influenced by model uncertainty.

The control format described in this report assumes that the nominal throttle setting is dictated by guidance and trajectory considerations and that some perturbations about this nominal setting are permitted so that dynamic pressure can be regulated.

It is possible to envision formats in which perturbations about the nominal throttle setting are not permitted. In such a case, the guidance system may command altitude changes to regulate dynamic pres-

sure in accordance with the throttle setting. The resolver concept presented in this report may still be used in conjunction with such alternative control formats.

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Table I. Geometric Characteristics of Winged-Cone Configuration

Wing:

Reference area (includes area projected to fuselage centerline), ft ²	3603.0
Aspect ratio	1.0
Span, ft	60.0
Leading-edge sweep angle, deg	76.0
Trailing-edge sweep angle, deg	0.0
Mean aerodynamic chord, ft	80.0
Airfoil section	Diamond
Airfoil thickness-to-chord ratio, percent	4.0
Incidence angle, deg	0.0
Dihedral angle, deg	0.0

Wing flap (elevon):

Area, each, ft ²	92.3
Chord (constant), ft	7.2
Inboard-section span location, ft	15.0
Outboard-section span location, ft	27.8

Vertical tail:

Exposed area, ft ²	645.7
Theoretical area, ft ²	1248.8
Span, ft	32.5
Leading-edge sweep angle, deg	70.0
Trailing-edge sweep angle, deg	38.1
Airfoil section	Diamond
Airfoil thickness-to-chord ratio, percent	4.0

Rudder:

Area, ft ²	161.4
Span, ft	22.8
Ratio of chord to vertical tail, percent	25.0

Axisymmetric fuselage:

Theoretical length, ft	200.0
Cone half-angle, deg	5.0
Cylinder radius (maximum), ft	12.9
Cylinder length, ft	12.9
Boattail half-angle, deg	9.0
Boattail length, ft	40.0
Moment reference center, ft	124.0

Table II. Aerodynamic Stability and Control Derivatives for Design Example

	Mach 6	Mach 10	Mach 15	Mach 20
C_{y_β}	-9.34×10^{-3}	-7.34×10^{-3}	-6.52×10^{-3}	-6.20×10^{-3}
$C_{y_{\delta r}}$	2.40×10^{-4}	1.30×10^{-4}	9.00×10^{-5}	7.00×10^{-5}
C_{l_β}	-7.00×10^{-4}	-4.70×10^{-4}	-3.40×10^{-4}	-2.70×10^{-4}
$C_{l_{\delta r}}$	7.00×10^{-5}	4.00×10^{-5}	3.00×10^{-5}	2.00×10^{-5}
$C_{l_{\delta a}}$	2.00×10^{-4}	1.40×10^{-4}	1.10×10^{-4}	1.00×10^{-4}
C_{l_r}	9.09×10^{-2}	5.62×10^{-2}	3.92×10^{-2}	3.11×10^{-2}
C_{l_p}	-7.93×10^{-2}	-5.23×10^{-2}	-3.79×10^{-2}	-2.90×10^{-2}
C_{n_β}	6.90×10^{-4}	-5.30×10^{-4}	-1.04×10^{-3}	-1.25×10^{-3}
$C_{n_{\delta r}}$	-3.30×10^{-4}	-1.80×10^{-4}	-1.30×10^{-4}	-1.00×10^{-4}
C_{n_r}	-6.90×10^{-1}	-4.91×10^{-1}	-4.10×10^{-1}	-3.78×10^{-1}
C_{n_p}	8.77×10^{-2}	5.42×10^{-2}	3.82×10^{-2}	3.07×10^{-2}

Table III. Aerodynamic Lift and Drag Coefficients for Design Example

	Mach 6	Mach 10	Mach 15	Mach 20
C_{L,α_0}	-1.329×10^{-3}	-1.215×10^{-3}	-1.251×10^{-3}	-1.285×10^{-3}
$C_{L,\alpha}$	1.359×10^{-2}	1.044×10^{-2}	9.200×10^{-3}	8.700×10^{-3}
C_{D,α_0}	9.931×10^{-3}	6.401×10^{-3}	3.767×10^{-3}	3.325×10^{-3}
$C_{D,\alpha}$	6.640×10^{-5}	-5.096×10^{-5}	-2.211×10^{-6}	-2.187×10^{-4}
C_{D,α^2}	2.695×10^{-4}	2.405×10^{-4}	2.449×10^{-4}	2.620×10^{-4}

Table IV. Trim Parameters for Four Hypersonic Cruise Conditions

Mach	h_0 , ft	α_0 , deg	T_0/W	V_0 , ft/sec	$1 - \frac{V_0^2}{Rg}$, g units
6	71 000	3.51	0.278	5 466	0.956
10	95 000	3.83	.217	9 626	.863
15	114 000	2.92	.151	15 094	.663
20	130 000	1.18	.106	22 097	.278

Table V. Frequency and Damping Parameters for Design Example

Equation	Parameter	Control	Parameter value	Period of oscillation, sec
(A26a), (A26b)	ζ_a	Aileron	0.70	2.1
	ω_a	Aileron	3.0 rad/sec	
(A12a), (A12b)	ζ_r	Rudder	0.70	2.1
	ω_r	Rudder	3.0 rad/sec	
(C7)	σ_t	Throttle	2.0 rad/sec	3.1
(5a), (5b), (5c)	σ_h	Altitude outer loop	0.1047 rad/sec	60
(5a), (5b), (5c)	ζ_h	Altitude outer loop	0.80	
	ω_h	Altitude outer loop	0.0698 rad/sec	90
(5a), (5b), (5c)	σ_y	Cross-range outer loop	0.0785 rad/sec	80
(5a), (5b), (5c)	ζ_y	Cross-range outer loop	0.80	
	ω_y	Cross-range outer loop	0.698 rad/sec	90

Table VI. Summary of Response Time Histories

Figure	Flight conditions (Mach number)	Input type
10	6, 10, 15, 20 ↓	Command: 2000-ft altitude increase
11		Command: 20 000-ft cross-range change
12		Command: 20 000-ft cross-range change, polar plot of load factor versus bank angle
13		Command: combined 2000-ft altitude increase and 20 000-ft cross-range change
14	↓ 15 6, 10, 15, 20 ↓	Command: 10° heading change
15		Command: 5000-ft descent, bank-to-dive versus pure descent
16		Command: 5000-ft descent, bank-to-dive
17		Disturbance: 30-knot headwind increase
18	↓ 10	Disturbance: 30-knot crosswind
19		Disturbance: 10-percent density pulse
20		Disturbance: 10-percent density pulse of various durations

Figure 1. Coordinate system and notation for body axes.

Figure 2. Basic elements of hypersonic control system.

Figure 3. Basic altitude loop proportional-integral-derivative (p-i-d) architecture.

Figure 4. Altitude and cross-range loops interconnected through resolver.

Figure 5. Resolver axis system and coordinate transformation.

Figure 6. Load factor limits in resolver coordinate system.

Figure 7. Load factor limits to prioritize altitude regulation over cross range.

Figure 8. Maximum bank angle used in turn-rate limit.

Figure 9. Hypersonic winged-cone configuration.

Figure 10. Simulated perturbation responses to 2000-ft altitude command change.

Figure 10. Concluded.

Figure 11. Simulated perturbation responses to 20 000-ft cross-range command change.

Figure 11. Continued.

Figure 11. Continued.

Figure 11. Concluded.

Figure 12. Polar plot of load factor versus bank angle time histories for 20 000-ft cross-range command change.

Figure 13. Simulated responses to combination of altitude and cross-range commands.

Figure 13. Continued.

Figure 13. Continued.

Figure 13. Concluded.

Figure 14. Simulated responses to a 10° heading-change command.

Figure 14. Continued.

Figure 14. Concluded.

Figure 15. Bank-to-dive maneuver compared with pure descent at Mach 15.

Figure 15. Continued.

Figure 15. Continued.

Figure 15. Concluded.

Figure 16. Responses to bank-to-dive maneuver at four hypersonic flight conditions.

Figure 16. Concluded.

Figure 17. Simulated perturbation responses to 30-knot headwind.

Figure 17. Concluded.

Figure 18. Cross-range deviation and sideslip responses to 30-knot crosswind.

Figure 19. Perturbation responses to negative 10-percent density pulse, 10 miles in length.

Figure 19. Continued.

Figure 19. Concluded.

Figure 20. Perturbation responses to negative 10-percent density pulses of various lengths at Mach 10.

Figure 20. Continued.

Figure 20. Concluded.

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13. ABSTRACT (Maximum 200 words) Piloting difficulties associated with conducting maneuvers in hypersonic flight are caused in part by the nonintuitive nature of the aircraft response and the stringent constraints anticipated on allowable angle-of-attack and dynamic pressure variations. This report documents an approach that provides precise, coordinated maneuver control during excursions from a hypersonic cruise flight path and observes the necessary flight condition constraints. The approach is to achieve specified guidance commands by resolving altitude and cross-range errors into a load factor and bank angle command by using a coordinate transformation that acts as an interface between outer- and inner-loop flight controls. This interface, referred to as a "resolver," applies constraints on angle-of-attack and dynamic pressure perturbations while prioritizing altitude regulation over cross range. An unpiloted test simulation, in which the resolver was used to drive inner-loop flight controls, produced time histories of responses to guidance commands and atmospheric disturbances at Mach numbers of 6, 10, 15, and 20. Angle-of-attack and throttle perturbation constraints, combined with high-speed flight effects and the desire to maintain constant dynamic pressure, significantly impact the maneuver envelope for a hypersonic vehicle. Turn-rate, climb-rate, and descent-rate limits can be determined from these constraints. Density variation with altitude strongly influences climb- and descent-rate limits and throttle modulation if dynamic pressure is to be maintained during vertical transitions between cruise flight conditions.				
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